sometimes answers need to be **discovered**
sometimes they need to be **invented**

Interface Determination of Two Shear Thinning Thermoplastic Composite Materials in the Extrusion Process

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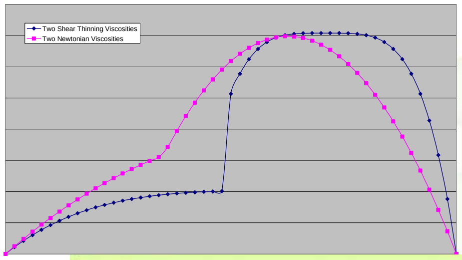

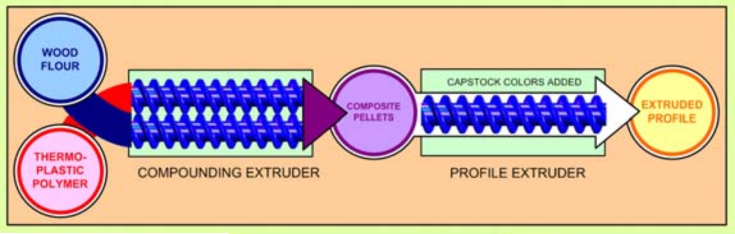

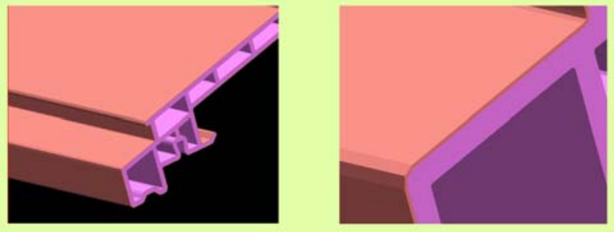

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Abstract

When two thermoplastic composite materials are extruded in a two dimensional die channel, their dimensions (interface and thickness) will be arranged based on viscosities and flow rates. Mathematical equations have been created to determine the interfacial dimension of two shear thinning materials with power law shear thinning. These equations have been applied to two dimensional extrusion die channels and the method will benefit optimal die design and the control of extrusion processes.

Introduction

Polymer processes (fiber, film and rigid lineal extrusions as well as coatings) often utilize co-extrusion. When two thermoplastic composite materials are co-extruded through a single die channel, the proportion of each material depends on each material's flow rate and viscosity. Controlling interface dimension and each material's thickness is critical and challenging. In the study of controlling these dimensions, the thermoplastic composite materials are considered viscous fluids.

The goal of this paper is to provide an easy-to-use method to predict the interface between two shear thinning thermoplastic materials in a co-extrusion die. Because the theory is based on two dimensional parallel Stokes flow, a user can create basic spread sheets to calculate the interface and thickness dimensions from geometry and fluid properties.

Background

Extensive research has been performed on the co-extrusion of thermoplastic composite materials since the 1970's. In the extrusion of two molten polymers through circular and rectangular pipes, experiment data shows a tendency for the lower viscosity polymer to migrate to the exterior of the higher viscosity polymer. This tendency appears to be influenced more by a difference in viscosity of the two polymers than by other properties.

Thermoplastic composite materials are viscous fluids with shear thinning behavior referred to as Non-Newtonian. This means that when the rate of shear increases in these fluids, shear viscosity decreases.

This shear thinning allows for greater migration of lower viscosity fluids to the exterior of the higher viscosity fluids. In the case of two dimensional die channels, these exteriors are higher shear regions and causes for greater shear thinning of the migrating fluid. This migration continues until equilibrium is reached resulting in the shear velocity profile. The amount of migration and eventual shear velocity profile determines the interface and thickness dimensions of the two fluids.

The following work focuses on how shear thinning impacts the shear velocity profiles of the co-extrusion process. The results are compared with Newtonian fluids, which are considered to be a special case of shear thinning behavior.

Governing Equations and Solutions

The extrusion die can be considered as a two dimensional parallel channel (see Figure 1), and the extruded materials in the channel can be considered as two viscous fluids. Since the flow speed is very slow and viscosity dominates, the governing equation for a steady flow is simplified to a Poiseuille flow

$$\frac{\partial}{\partial \boldsymbol{x}} \mu \frac{\partial \boldsymbol{u}}{\partial \boldsymbol{x}} - \frac{\boldsymbol{dp}}{\boldsymbol{dz}} = 0 \tag{1}$$

Most thermoplastic composite materials have shear thinning behavior. A typical shear thinning power law model is

$$\mu = \mu_{\boldsymbol{o}} \left(\frac{\partial \boldsymbol{u}}{\partial \boldsymbol{x}} \right)^{\boldsymbol{n}-1} \tag{2}$$

and for Newtonian flow

$$\mu = \mu_{\boldsymbol{o}}$$

where

μ is the viscosity

$\mu_{\boldsymbol{o}}$ is constant or zero shear viscosity

$\frac{\partial \boldsymbol{u}}{\partial \boldsymbol{x}}$ is shear rate

$0 \le \boldsymbol{n} < 1$

Since the flow is two-fluid flow, the flow properties in two regions are different.

The viscosity in region 1 is

$$\mu_1 = \mu_{\boldsymbol{o}1} \left(\frac{\partial \boldsymbol{u}_1}{\partial \boldsymbol{x}} \right)^{\boldsymbol{n}_1 - 1}$$

The viscosity in region 2 is

$$\mu_2 = \mu_{\boldsymbol{o}2} \left(\frac{\partial \boldsymbol{u}_2}{\partial \boldsymbol{x}} \right)^{\boldsymbol{n}_2 - 1}$$

where

$\mu_{\boldsymbol{o}1}$ is constant viscosity in region 1

$\mu_{\boldsymbol{o}2}$ is constant viscosity in region 2

According to no-slip boundary condition,

$$\begin{aligned} &\boldsymbol{at} \quad \boldsymbol{x} = 0 \quad \boldsymbol{u}_1 = 0 \\ &\boldsymbol{at} \quad \boldsymbol{x} = \boldsymbol{R}_{\boldsymbol{o}} \quad \boldsymbol{u}_2 = 0 \end{aligned} \tag{3}$$

Assume there is no penetration between two fluids. At the interface of fluid 1 and fluid 2, the velocities and shear stresses are continuous. Thus

$$\boldsymbol{at} \quad \boldsymbol{x} = \boldsymbol{R}_1 \quad \begin{cases} \boldsymbol{u}_1 = \boldsymbol{u}_2 \\ \mu_1 \frac{\partial \boldsymbol{u}_1}{\partial \boldsymbol{x}} = \mu_2 \frac{\partial \boldsymbol{u}_2}{\partial \boldsymbol{x}} \end{cases} \tag{4}$$

where

$\boldsymbol{R}_1$ is the interface location of two materials.

Solutions of Newtonian Fluids

The general solution for a Newtonian fluid (1) is

$$\boldsymbol{u} = \beta \frac{\boldsymbol{x}^2}{2\mu_{\boldsymbol{o}}} + \frac{\boldsymbol{C}_1}{\mu_{\boldsymbol{o}}} \boldsymbol{x} + \boldsymbol{C}_2 \tag{5}$$

where

$\beta = \frac{\boldsymbol{dp}}{\boldsymbol{dz}}$ is the pressure gradient

In the first material or μ_1 region, the solution is

$$\boldsymbol{u}_1 = \beta \frac{\boldsymbol{x}^2}{2\mu_{\boldsymbol{o}1}} + \frac{\boldsymbol{C}_{11}}{\mu_{\boldsymbol{o}1}} \boldsymbol{x} + \boldsymbol{C}_{12} \tag{6}$$

In the second material, or μ_2 region, the solution is

$$\boldsymbol{u}_2 = \beta \frac{\boldsymbol{x}^2}{2\mu_{\boldsymbol{o}2}} + \frac{\boldsymbol{C}_{21}}{\mu_{\boldsymbol{o}2}} \boldsymbol{x} + \boldsymbol{C}_{22} \tag{7}$$

Based on no-slip boundary conditions (3), and the interface conditions (4), the constants in (6) and (7) become:

$$\begin{aligned} C_{12} &= 0 \\ C_{11} &= \frac{\beta}{2} \frac{m\left(R_1^2 - R_o^2\right) - R_1^2}{R_1 - m\left(R_1 - R_o\right)} \\ C_{21} &= C_{11} \\ \boldsymbol{C}_{22} &= -\frac{\beta \boldsymbol{R}_0}{2\mu_{\boldsymbol{o}2}} - \frac{\boldsymbol{C}_{21}}{\mu_{\boldsymbol{o}2}} \boldsymbol{R}_0 \end{aligned} \tag{8}$$

where

$$\boldsymbol{m} = \frac{\mu_{o1}}{\mu_{o2}}$$

The velocity solutions are

$$\boldsymbol{u_1} = \frac{\beta \boldsymbol{x}}{2\mu_{o1}}\left[\boldsymbol{x} + \frac{\boldsymbol{m}\left(\boldsymbol{R_1^2} - \boldsymbol{R_0^2}\right) - \boldsymbol{R_1^2}}{\boldsymbol{R_1} - \boldsymbol{m}\left(\boldsymbol{R_1} - \boldsymbol{R_0}\right)}\right]$$

$$\left(0 \le x \le R_1\right) \qquad (9)$$

$$\boldsymbol{u_2} = \frac{\beta}{2\mu_{o2}}\left[\left(\boldsymbol{x}^2 - \boldsymbol{R_0^2}\right) + \left(\boldsymbol{x} - \boldsymbol{R_0}\right)\frac{\boldsymbol{m}\left(\boldsymbol{R_1^2} - \boldsymbol{R_0^2}\right) - \boldsymbol{R_1^2}}{\boldsymbol{R_1} - \boldsymbol{m}\left(\boldsymbol{R_1} - \boldsymbol{R_0}\right)}\right]$$

$$\left(R_1 < x \le R_0\right) \qquad (10)$$

If $m = 1$, the two materials are identical, (9) and (10) are the same. If $m \neq 1$, the velocity profiles will also depend on their flow rates. Figure 2 shows the velocity profiles of fluids with identical viscosity vs. different viscosities.

Now, the question is how to determine the interface $\boldsymbol{R_1}$. In reality, the extrusion flow rate is a control parameter. By integrating the flow velocities, the volume flow rates can be obtained.

In the first material (μ_1 region), the volume flow rate is

$$\boldsymbol{Q_1} = \int_0^{\boldsymbol{R_1}} \boldsymbol{u_1 dx} = \frac{\beta}{2\mu_{o1}}\left[\frac{\boldsymbol{R_1^3}}{3} + \frac{\boldsymbol{R_1^2}}{2}\,\frac{\boldsymbol{m}\left(\boldsymbol{R_1^2} - \boldsymbol{R_0^2}\right) - \boldsymbol{R_1^2}}{\boldsymbol{R_1} - \boldsymbol{m}\left(\boldsymbol{R_1} - \boldsymbol{R_0}\right)}\right] (11)$$

In the second material (μ_2 region), the volume flow rate is

$$Q_2 = \int_{R_1}^{R_0} u_2 dx = \frac{\beta}{2\mu_{o2}}$$

$$\left[\begin{array}{l} \dfrac{R_0^3 - R_1^3}{3} - R_0^2\left(R_0 - R_1\right) + \\ \left(\dfrac{R_0^2 - R_1^2}{2} - R_0\left(R_0 - R_1\right)\right)\dfrac{m\left(R_1^2 - R_0^2\right) - R_1^2}{R_1 - m\left(R_1 - R_0\right)} \end{array}\right] \qquad (12)$$

When (11) and (12) are combined, there are two unknowns, R_1 and β.

The interface can be determined by following dimensionless equation:

$$m\gamma = -\frac{\eta^2}{\left(1-\eta\right)^2}\,\frac{\eta^2\left(m-1\right) - 3m + 2m\eta}{\eta^2\left(m-1\right) + m + \eta\left(4-2m\right)}$$

(9)

(13)

where

$$\eta = \frac{R_1}{R_0}\ , \gamma = \frac{Q_1}{Q_2} \text{ and } \boldsymbol{m} = \frac{\mu_{o1}}{\mu_{o2}}$$

Solution of Power Law Shear Thinning Fluids

The general solution for a power law shear thinning fluid (2) is

$$\boldsymbol{u} = \frac{\mu_o}{\boldsymbol{N}\beta}\left[\frac{\beta \boldsymbol{x}}{\mu_o} + \frac{\boldsymbol{C_1}}{\mu_o}\right]^{\boldsymbol{N}} + \boldsymbol{C_2}$$

(14)

where

$$\boldsymbol{N} = 1 + \frac{1}{\boldsymbol{n}}$$

In the first fluid, or μ_1 region, the velocity solution is

$$\boldsymbol{u} = \frac{\mu_{o1}}{\boldsymbol{N_1}\beta}\left[\frac{\beta \boldsymbol{x}}{\mu_{o1}} + \frac{\boldsymbol{C_{11}}}{\mu_{o1}}\right]^{\boldsymbol{N_1}} + \boldsymbol{C_{12}}$$

(15)

In the second fluid, or μ_2 region, the velocity solution is

$$\boldsymbol{u} = \frac{\mu_{o2}}{\boldsymbol{N_2}\beta}\left[\frac{\beta \boldsymbol{x}}{\mu_{o2}} + \frac{\boldsymbol{C_{21}}}{\mu_{o2}}\right]^{\boldsymbol{N_2}} + \boldsymbol{C_{22}} \qquad (16)$$

where

$$\boldsymbol{N}_1 = 1 + \frac{1}{\boldsymbol{n}_1}$$

$$\boldsymbol{N}_2 = 1 + \frac{1}{\boldsymbol{n}_2}$$

and

$\boldsymbol{n}_1$, $\boldsymbol{n}_2$ are the fluid index.

Based on no-slip boundary conditions (3), and the interface conditions (4), the constants in (15) and (16) become:

$$\boldsymbol{C}_{12} = -\frac{\mu_{o1}}{\boldsymbol{N}_1\beta}\left[\frac{\boldsymbol{C}_{11}}{\mu_{o1}}\right]^{\boldsymbol{N}_1}$$

$$\boldsymbol{C}_{21} = \boldsymbol{C}_{11}$$

$$\boldsymbol{C}_{22} = -\frac{\mu_2}{\boldsymbol{N}\beta}\left[\frac{\beta\boldsymbol{R}_o}{\mu_2} + \frac{\boldsymbol{C}_{21}}{\mu_2}\right]^{\boldsymbol{N}_2}$$

$$[\beta R_1 + C_{11}]^{N_1} - [C_{11}]^{N_1} = \frac{N_1}{N_2}\frac{m^{N_1-1}}{\mu_{o2}{}^{N_2-N_1}}\left\{[\beta R_1 + C_{11}]^{N_2} - [\beta R_o + C_{11}]^{N_2}\right\} \quad (17)$$

where

$$\boldsymbol{m} = \frac{\mu_{o1}}{\mu_{o2}}$$

The general solutions of two shear thinning fluids are

$$\boldsymbol{u}_1 = \frac{\mu_{o1}}{\boldsymbol{N}_1\beta}\left[\frac{\beta\boldsymbol{x}}{\mu_{o1}} + \frac{\boldsymbol{C}_{11}}{\mu_{o1}}\right]^{\boldsymbol{N}_1} - \frac{\mu_{o1}}{\boldsymbol{N}_1\beta}\left[\frac{\boldsymbol{C}_{11}}{\mu_{o1}}\right]^{\boldsymbol{N}_1}$$

$$(0 \le \boldsymbol{x} \le \boldsymbol{R}_1) \quad (18)$$

$$\boldsymbol{u}_2 = \frac{\mu_{o2}}{\boldsymbol{N}_2\beta}\left[\frac{\beta\boldsymbol{x}}{\mu_{o2}} + \frac{\boldsymbol{C}_{11}}{\mu_{o2}}\right]^{\boldsymbol{N}_2} - \frac{\mu_{o2}}{\boldsymbol{N}_2\beta}\left[\frac{\beta\boldsymbol{R}_0}{\mu_{o2}} + \frac{\boldsymbol{C}_{11}}{\mu_{o2}}\right]^{\boldsymbol{N}_2}$$

$$(\boldsymbol{R}_1 < \boldsymbol{x} \le \boldsymbol{R}_0) \quad (19)$$

A special case, if $m = 1$ and $\boldsymbol{N}_1 = \boldsymbol{N}_2 = \boldsymbol{N}$, (7) and (8) are the same and

$$\boldsymbol{C}_{11} = -\frac{\beta\boldsymbol{R}_o}{2}$$

Thus, in order to have a solution, $\boldsymbol{N}$ is required to be an even number in above case.

In this case, the two materials can be considered as one material as

$$\boldsymbol{u} = \frac{\mu_{o1}}{4\beta}\left[\frac{\beta\boldsymbol{x}}{\mu_{o1}} - \frac{\beta\boldsymbol{R}_o}{2\mu_{o1}}\right]^{N_1} - \frac{\mu_{o1}}{4\beta}\left[-\frac{\beta\boldsymbol{R}_o}{2\mu_{o1}}\right]^{N_1}$$

Compare to the solution of one Newtonian fluid:

$$\boldsymbol{u} = \frac{\beta\boldsymbol{x}}{2\mu_{o1}}[\boldsymbol{x} - \boldsymbol{R}_o]$$

The velocity profiles of one Newtonian and one Power law shear thinning (N=4) is shown in Figure 3.

When shear rate increases, the viscosity decreases, and flow profile is close to a plug flow. This is because the viscosity decreases in the high shear region, near the wall.

If $m \ne 1$ for two power law shear thinning fluids, the velocity profile will not be smooth. Figure 4 shows the velocity profiles of power law shear thinning fluids with identical constant viscosity vs. different constant viscosities.

Now, the problem is how to determine the interface $\boldsymbol{R}_1$ of two power law shear thinning fluids. In the extrusion process, the extrusion flow rate is the control parameter. By integrating the flow velocities, the flow rates are obtained:

In region one:

$$Q_1 = \int_0^{R_1} u_1 dx = \frac{1}{N_1 (N_1+1) \beta^2 \mu_{o1}^{N_1-1}}$$
$$\left\{ [\beta R_1 + C_{11}]^{N_1+1} - C_{11}^{N_1+1} - (N_1+1)\beta R_1 C_{11}^{N_1} \right\}$$
(20)

In region two:

$$Q_2 = \int_{R_1}^{R_0} u_2 dx = \frac{1}{N_2 (N_2+1) \beta^2 \mu_{o2}^{N_2-1}}$$
$$\left\{ \begin{matrix} [\beta R_o + C_{11}]^{N_2+1} - [\beta R_1 + C_{11}]^{N_2+1} - \\ (N_2+1)(\beta R_o + C_{11})^{N_2} (R_o - R_1) \end{matrix} \right\}$$
(21)

In (17), (20) and (21), there are three unknowns in three equations, $\boldsymbol{C}_{11}$, R_1 and β,

$$\boldsymbol{f}_{\boldsymbol{o}}(\boldsymbol{C}_{11}, \boldsymbol{R}_1, \beta) = 0$$

$$\boldsymbol{Q}_1 = \boldsymbol{f}_1(\boldsymbol{C}_{11}, \boldsymbol{R}_1, \beta)$$

$$\boldsymbol{Q}_2 = \boldsymbol{f}_2(\boldsymbol{C}_{11}, \boldsymbol{R}_1, \beta)$$

The interface can be determined by the following dimensionless equations:

$$[\eta + \boldsymbol{c}]^{\boldsymbol{N}_1} - [\boldsymbol{c}]^{\boldsymbol{N}_1} = \boldsymbol{M} \left\{ [\eta + \boldsymbol{c}]^{\boldsymbol{N}_2} - [1 + \boldsymbol{c}]^{\boldsymbol{N}_2} \right\}$$
(22)

$$\gamma M = \frac{(N_2+1)}{(N_1+1)}$$
$$\frac{(\eta + c)^{N_1+1} - c^{N_1+1} - 5\eta c^{N_1}}{(1+c)^{N_2+1} - (\eta + c)^{N_2+1} - 5(1+c)^{N_2}(1-\eta)}$$
(23)

where

$$\eta = \frac{\boldsymbol{R}_1}{\boldsymbol{R}_0} \qquad 0 \le \eta \le 1$$

$$\gamma = \frac{\boldsymbol{Q}_1}{\boldsymbol{Q}_2}$$

$$\boldsymbol{m} = \frac{\mu_1}{\mu_2}$$

$$\boldsymbol{c} = \frac{\boldsymbol{C}_{11}}{\beta \boldsymbol{R}_{\boldsymbol{o}}}$$

$$\boldsymbol{M} = \frac{\boldsymbol{N}_1}{\boldsymbol{N}_2} \frac{\boldsymbol{m}^{\boldsymbol{N}_1 - 1} (\beta \boldsymbol{R}_{\boldsymbol{o}})^{\boldsymbol{N}_2 - \boldsymbol{N}_1}}{\mu_{\boldsymbol{o}2}^{\boldsymbol{N}_2 - \boldsymbol{N}_1}}$$

In (22) and (23), dimensionless parameter η and $\boldsymbol{c}$ are the unknowns. Those unknowns may not be solved by analytic method, but can be solved by a numerical method.

The interface $(R_1 \text{ or } \eta)$ is the function of the ratios of the flow rates and fluid viscosities, and also depends on the index of power law shear thinning.

Solution of Combined Newtonian and Power Law Shear Thinning Fluids

The Newtonian fluid can be considered as a special case of power law shear thinning when fluid index $\boldsymbol{n} = 1$. Therefore, the general equations of fluid velocities (18) and (19), and interface dimensions (22) and (23) already include the solutions of combined Newtonian and power law shear thinning fluids.

Results and Discussion

The general solutions for solving the interface between two power law shear thinning fluids have been created. Once the flow rates and fluids' properties are determined, the velocity profiles or the interface can be calculated.

In order to illustrate the difference of Newtonian and shear thinning fluids, a simplified model of power law is made: assume the co-extruded two power law shear thinning fluids have both index $\boldsymbol{n} = 1/3$. Four special cases are discussed, and the difference between Newtonian flows and power law shear thinning flows is compared.

Case Studies

Case 1

Conditions: $m = 1$, and any γ, the interface can be anywhere depending on the flow rates. If $m = 1$, the constant viscosities are the same, and two fluids can be considered as one fluid flow. Thus, the interface only depends on the flow rates of two fluids. In this case, Interfaces can be anywhere depending on flow rate ratio γ (see Figure 3).

For example, if $\gamma = 1$, the interface for both Newtonian and shear thinning fluids is located in the middle ($\eta = 0.5$). If $\gamma = 3$, the interface for Newtonian fluid is at $\eta = 0.67$ and for shear thinning is at $\eta = 0.74$. If $\gamma = 5$, the interface for Newtonian fluid is at $\eta = 0.70$ and for shear thinning is at $\eta = 0.82$. The velocity profiles for both Newtonian and shear thinning fluids are continuous, smooth and also seem symmetric because their viscosities are same and continuous.

Case 2

Conditions: $m = 5$, and $\gamma = 1$, the interface for Newtonian fluid flows is located at $\eta = 0.57$, and for power law shear thinning flow is at $\eta = 0.615$ (see Figure 5). In this case, the higher viscosity fluid tends to flow slower and lower viscosity component tries to flow faster. In order to keep the same flow rate, the interface migrates toward lower viscosity fluid.

Even though the flow rates are the same, the interface locations for both Newtonian and shear thinning fluids are off center. The velocity profiles are not symmetric because there is a sudden change of viscosity at the interface.

Case 3

Conditions: $m = 5$, and $\gamma = 5$, both viscosity ratio and flow rate ratio are 5, and the interface for Newtonian flow is at $\eta = 0.8$, and for shear thinning flow is at $\eta = 0.82$ (see Figure 6). In this case, the higher viscosity fluid (in region one) has a higher flow rate with the interface towards the lower viscosity component in region two.

When higher viscosity fluid has higher flow rate, the majority of the channel's space is occupied by the higher viscosity fluid, which squeezes the interface closer to the wall.

Case 4

Conditions: $m = 5$, and $\gamma = 0.2$, when viscosity ratio is 5 and flow rate ratio is 0.2, the interface for Newtonian fluid flow is at $\eta = 0.36$ (see Figure 7) and power law shear thinning flow is at $\eta = 0.49$. In this case, higher viscosity fluid has the lower flow rate.

Case 4 is the opposite situation of Case 3. When two fluids have significantly different viscosities, in order to balance the interface location, lower viscosity fluid should have higher flow rate.

Figures

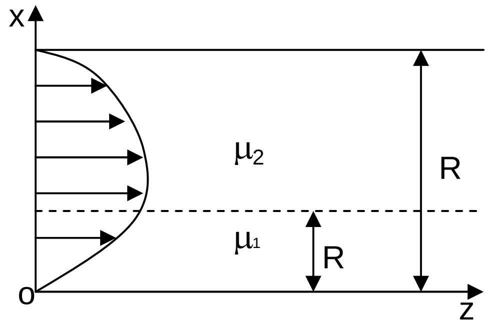

Figure 1: Flow diagram

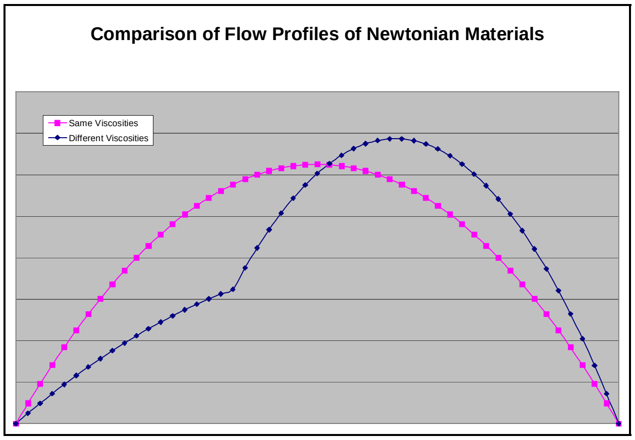

Figure 2: Comparison of two Newtonian flow profiles with identical and different viscosities.

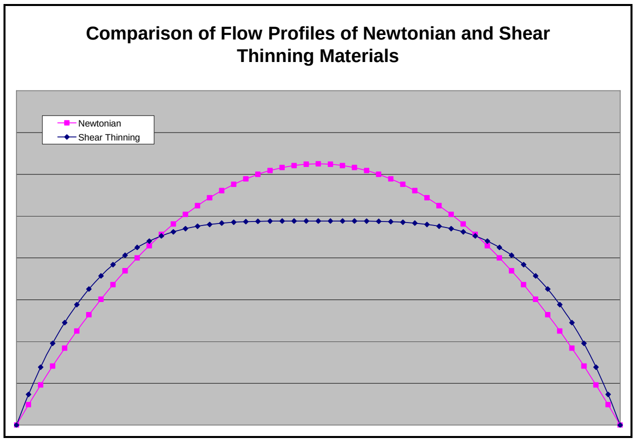

Figure 3: The comparison of velocity profiles of Newtonian and shear thinning materials. The shear thinning profile is more like plug flow.

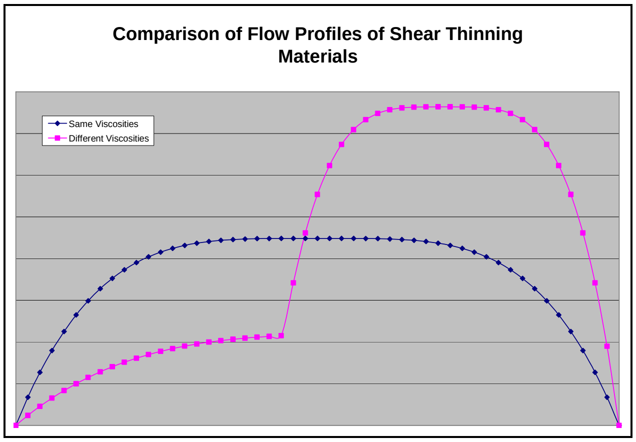

Figure 4: Comparison of flow profiles of two power law shear thinning fluids with identical constant viscosity vs. different constant viscosities.

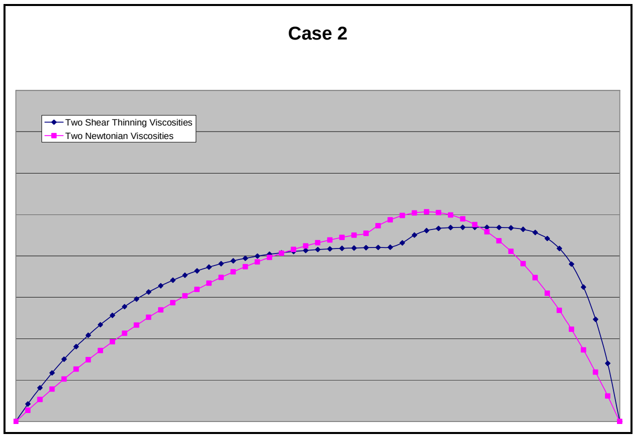

Figure 5: For velocity profiles of Case 2, interface for viscous flow is at 0.57 and shear thinning flow is at 0.615.

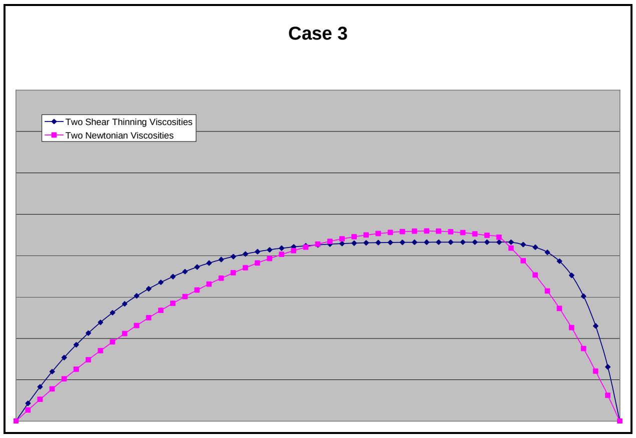

Figure 6: For velocity profiles of Case 3, the interface for viscous flow is at 0.80, and shear thinning flow is at 0.82.

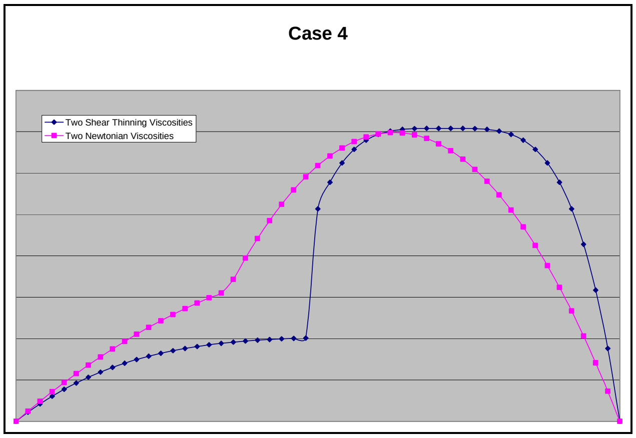

Figure 7: For velocity profiles of Case 4, the interface for viscous flow is at 0.36, and shear thinning flow is at 0.49.

Conclusions

By using the equations created above, interface and thickness dimensions for two thermoplastic composite materials co-extruded in a two dimensional die channel can be predicted. These equations can be used to create basic spread sheets (tools) to calculate the interface and thickness dimensions from geometry and fluid properties.

As is described in the above case studies, flow rates and viscosities (ratios) affect the interface and thickness dimensions. Process inputs (throughput, temperatures and shear rates) directly affect flow rate and viscosity and should be explicitly designed and controlled to achieve intended dimensions.

Key Words:

Shear Thinning, Interface, Viscosity, Co-Extrusion, Two Phase Flow.

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